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AN APPROXIMATE NUMERICAL SCHEME FOR THE THEORY OF CAVITY FLOWS PAST OBSTACLES OF ARBITRARY PROFILE

by

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Recently an exact theory for the cavity flow past an obstacle of arbitrary profile at an arbitrary cavitation number has been developed by adopting a free-streamline wake model. The analysis in this general case leads to a set of two functional equations for which several numerical methods have been devised; some of these methods have already been successfully carried out for several typical cases on a high speed electronic computer.

In this paper an approximate numerical scheme, somewhat like an engineering principle, is introduced which greatly shortens the computation of the dual functional equations while still retaining a high degree of accuracy of the numerical result. With such drastic simplification, it becomes feasible to carry out this approximate numerical scheme even with a hand computing machine.

INTRODUCTION

A free-streamline wake model has been recently introduced by one of the authors (Wu [1][3]) to treat the fully and partially developed cavity flow or wake flow past an inclined flat plate. The exact solution of this problem can be expressed in a closed form for the entire range of the cavitation number as the cavity decreases from an infinite length to a vanishingly small leading edge bubble as the cavity disappears.

This theory has been further generalized by the authors [2] to consider the cavity flow past an obstacle of arbitrary profile at an arbitrary cavitation number. The analysis in this general case leads to a set of two functional integral equations for which the numerical methods seem to be the only means of obtaining an accurate solution. For this purpose several numerical methods have already been developed; they fall in the following categories: (i) an integral iteration method, (ii) a differential perturbation, and (iii) discretization of the slope function, all iterations or perturbations being referred with respect to a known basic flow. Of these methods the integral iteration seems to be the most general and versatile, as has been exhibited by the results of its applications in several typical cases. This iteration program is relatively simple and straightforward for a high speed electronic computer, and the accuracy of the result as high as four significant figures can be easily achieved.

The purpose of this paper is to introduce an even simpler approximate numerical scheme, which appears in a way like an engineering principle, for calculation of the dual functional equations. As shown in several actual applications, this approximate method has greatly shortened the computation of the dual functional equations while still retaining a high degree of

3. Numbers in brackets designate References at end of paper.

accuracy of the numerical result. As a specific example, the circular arc hydrofoil of total arc angle 16° has been evaluated by the original integral iteration and this proposed approximate numerical method. For each prescribed cavitation number and angle of attack and for a designated interval size for the numerical integration, the computer time for a single iteration in solving the dual functional equation by the integral iteration method on an IBM-7090 computer has been recorded to take about 30 seconds. By adopting the approximate scheme, the corresponding computer time for a single iteration using the same integration program on the same computer is reduced to 3 seconds. This remarkable saving in time and effort is very desirable, especially when there are a large number of cases to be computed. Furthermore, with such drastic simplification, this approximate numerical scheme is considered to be quite feasible even for a hand computing machine. It is in this sense that this method may serve its engineering purposes in future applications.

THE CAVITY FLOW THEORY

The cavity flow theory for which this approximate numerical scheme is to be proposed has been presented in an earlier paper [2] . In order to facilitate the subsequent discussion and to make this work self-contained, we summarize this theory in the following.

The problem in question is the two-dimensional flow of an incompressible fluid past a solid obstacle, forming a wake or cavity bounded laterally by free-streamlines (see Figure 1). The detachment points A and B at which the free streamlines separate from the body may be either fixed or of the type of smooth detachment; in the latter case they are not known á priori. The wetted surface of body may be expressed parametrically in terms of the rectangular coordinates

$$x = x(s), \qquad y = y(s), \qquad \text{for} \quad 0 \leq s \leq S, \tag{1a}$$

where S is the total arc length of the wetted surface; and the inclination angle of the body surface with the x-axis is

$$\beta(s) = \arctan\left(\frac{dy}{ds}\Big/\frac{dx}{ds}\right), \qquad 0 \leq s \leq S. \tag{1b}$$

According to this wake model, the flow is approximately described by an equivalent potential flow past the body with an infinitely long wake which consists of a near-wake of constant under-pressure $p = p_c$ (as shown in Fig. 1, the near-wake is bounded laterally by free-streamlines AC and BC') and a far-wake that trails further downstream (bounded by streamlines CI and C'I) along which the pressure p increases from p_c to p_∞ as the point I at infinity is approached.

In terms of the complex variable $z = x + iy$, and the complex potential $f = \phi + i\psi$, we have the complex velocity

$$w = \frac{df}{dz} = u - iv = q\, e^{-i\theta} \tag{2}$$

which satisfies the following boundary conditions:

$$w = U\, e^{-i\alpha} \qquad \text{at} \quad z = \infty, \tag{3a}$$

$$q \equiv |w| = q_c = 1 \qquad \text{along AC and BC}', \tag{3b}$$

$$\begin{aligned} \theta \equiv -\arg w &= \beta(s) + \pi \qquad \text{along AD,} \quad 0 \leq s \leq s_o, \\ &= \beta(s) \qquad\quad\; \text{along DB,} \quad s_o \leq s \leq S, \end{aligned} \tag{3c}$$

where D denotes the stagnation point. In Eq.(3b) the constant speed q_c along the cavity boundary is normalized to unity, hence from the Bernoulli equation

$$p + \frac{1}{2}\rho q^2 = p_\infty + \frac{1}{2}\rho U^2 = p_c + \frac{1}{2}\rho q_c^2 \tag{4}$$

we have

$$U = (1 + \sigma)^{-1/2} \tag{5a}$$

where

$$\sigma \equiv (p_\infty - p_c)/(\tfrac{1}{2}\rho U^2), \tag{5b}$$

σ being the wake under-pressure coefficient, or the cavitation number for cavity flows. The description of the model is completed with the assumption that both f and w assume respectively the same value at the points C and C' and that the free streamlines CI and C'I form a branch slit of undetermined shape in the w-plane.

It is convenient to introduce the parametrization by which the solution is expressed in terms of an analytic function in the lower half unit circle of a t-plane, with the circular arc corresponding to the constant pressure free boundary and the diameter to the wetted surface in such a way that this parameter plane becomes the hodograph as the arbitrary wetted surface is degenerated into a flat plate. Thus the solution is best represented in the following parametric form

$$f(t) = \frac{At^2}{(t-t_0)(t-\bar{t}_0)(t-t_0^{-1})(t-\bar{t}_0^{-1})}, \tag{6}$$

$$w(t) = t \exp\left\{-\frac{(1-t^2)}{\pi}\int_{-1}^{1}\frac{\beta(\tau)d\tau}{(\tau-t)(\tau t-1)}\right\}, \tag{7}$$

where A is a real constant, t_0 is the undetermined image point of $z = \infty$, and $\beta(\tau)$ represents the inclination angle at the point $t = \tau$ on the real t-axis. That Eqs. (6) and (7) provide the required solution can be readily established as follows. From the local conformal behavior of the streamlines near the point of infinity (or $t = t_0$) and the stagnation point ($t = 0$), it follows that f must have a simple pole at $t = t_0$ and f is of order t^2 as $t \rightarrow 0$. The remaining three poles of f at $\bar{t}_0$, $1/t_0$, $1/\bar{t}_0$ are the reflections of the pole at t_0 into the real t-axis and then into the unit circle $|t| = 1$ so that the real diameter and the circle $|t| = 1$ are ensured to be the bounding streamlines on which $\psi = 0$. Furthermore, this model requires f to be a one-valued function in the half circle of the t-plane. This establishes Equation (6).

We next notice that on the circle $|t| = 1$, the exponent in the curly bracket of Eq.(7) is purely imaginary and hence Condition (3b) is satisfied. Furthermore, when β is everywhere continuous $t = 0$ is the only stagnation point; and as an interior point t approaches a boundary point τ on the real diameter, it is readily seen from Eq.(7) that

$$\begin{aligned} \theta = -\arg w &= \beta(\tau) + \pi \qquad \text{for} \quad -1 < \tau < 0, \\ &= \beta(\tau) \qquad\qquad \text{for} \quad 0 < \tau < 1. \end{aligned}$$

Condition (3c) is therefore satisfied if a one-to-one relationship between the arc length s and the point τ on the real t-axis can be established. To determine this relationship, we first complete the solution by providing the physical z-plane by integration

$$z(t) = \int_{-1}^{t} \frac{1}{w(t)} \frac{df}{dt} dt. \tag{8a}$$

In particular, on the body surface (t real and $-1 < t < 1$),

$$z(t) = \int_{-1}^{t} e^{i\beta(t)} \exp\left\{ \frac{(1-t^2)}{\pi} \int_{-1}^{*1} \frac{\beta(\tau) d\tau}{(\tau - t)(\tau t - 1)} \right\} \frac{df}{dt} \frac{dt}{t} \tag{8b}$$

where * above the integral sign signifies the Cauchy principal value. Since $dz = e^{i\beta} ds$, and since for real t, $-1 < t < 1$,

$$\int_{-1}^{*1} \frac{d\tau}{(\tau - t)(\tau t - 1)} = 0,$$

it follows that the arc length $s(t)$ along the body surface is

$$s(t) = \int_{-1}^{t} \exp\left\{ \frac{(1-t^2)}{\pi} \int_{-1}^{1} \frac{\beta(\tau) - \beta(t)}{(\tau - t)(\tau t - 1)} d\tau \right\} \frac{df}{dt} \frac{dt}{t}. \tag{9}$$

Finally, application of condition (3a) to Eq.(7) yields

$$U e^{-i\alpha} = t_o \exp\left\{-\frac{(1-t_o^2)}{\pi}\int_{-1}^{1}\frac{\beta(\tau)d\tau}{(\tau-t_o)(\tau t_o-1)}\right\}. \tag{10}$$

For the direct problems, U, α are prescribed and the inclination angle β is given as a function of the arc length s instead of τ (see Eq.(1b)), then Eqs. (9) and (10) provide a set of two functional integral equations for the complex parameter t_o and the arbitrary real function $\beta(s(t;\, t_o))$. Finally, the real coefficient A is fixed by the physical scale which requires

$$s(1) = S \qquad \text{at} \qquad t = 1. \tag{11}$$

In case the detachment at A, B is of the smooth type, their location can be determined by imposing in addition the finite curvature condition that the curvature of the free streamline at a smooth-separation point must not be infinite. This condition can be written as

$$\lim_{t\to\pm 1}\left\{\frac{1}{t} + \frac{1}{\pi}\frac{d}{dt}\int_{-1}^{1}\left(\frac{1}{\tau-t} - \frac{t}{\tau t-1}\right)\beta(\tau)d\tau\right\} = 0. \tag{12}$$

With $\beta(s(t;t_o))$, t_o and A so determined (from Eqs. (9) - (12)), the lift L and drag D are given by (see Reference [2])

$$D + iL = \pi\rho\left\{\frac{1}{U}BG - U\bar{B}\bar{G}\right\}, \tag{13a}$$

where

$$B = \frac{A t_o^3 \bar{t}_o}{(t_o-\bar{t}_o)(1-t_o^2)(1-t_o\bar{t}_o)}, \tag{13b}$$

$$G(t_o) = \frac{1}{t_o} + \frac{1}{\pi}\int_{-1}^{1}\left[\frac{1}{(\tau-t_o)^2} + \frac{1}{(\tau t_o-1)^2}\right]\beta(\tau)d\tau. \tag{13c}$$

The pressure distribution at the solid surface (t real, $-1 < t < 1$) is given parametrically by s(t) of Eq.(9) and the following pressure coefficient

$$C_p(t) \equiv \frac{p-p_\infty}{\frac{1}{2}\rho U^2} = 1-(1+\sigma)w\bar{w} = 1-(1+\sigma)t^2\exp\left[\frac{2(1-t^2)}{\pi}\int_{-1}^{1}\frac{\beta(t)-\beta(\tau)}{(\tau-t)(\tau t-1)}d\tau\right]. \tag{14}$$

THE ORIGINAL INTEGRAL ITERATION METHOD

Several numerical methods of solution for the above dual functional equations (9), (10) have been proposed previously in Reference [2]. Of these methods perhaps the most versatile and suitable for the general purpose is the so-called "integral iteration method" which can be summarized as follows. Equations (9) and (10) may be rewritten for the iteration scheme as

$$\frac{1}{A^{(n)}}\, s^{(n)}(t;t_o^{(n)}) = \int_{-1}^{t} \mathcal{E}\left[\beta(s^{(n-1)}(t;t_o^{(n-1)}))\right] \frac{d}{dt}\left[\frac{f(t;t_o^{(n-1)})}{A^{(n-1)}}\right]\frac{dt}{t}, \tag{15}$$

$$t_o^{(n)} = U e^{-i\alpha}\, \mathcal{F}\left[\beta(s^{(n-1)}(t;t_o^{(n-1)}))\right], \tag{16}$$

for $n = 1, 2, 3, \ldots$, where

$$\mathcal{E}\left[\beta(s(t;t_o))\right] \equiv \exp\left\{\frac{(1-t^2)}{\pi}\int_{-1}^{1}\frac{\beta(\tau)-\beta(t)}{(\tau-t)(\tau t-1)}\,d\tau\right\}, \tag{17}$$

$$\mathcal{F}\left[\beta(s(t;t_o))\right] \equiv \exp\left\{\frac{(1-t_o^2)}{\pi}\int_{-1}^{1}\frac{\beta(\tau)\,d\tau}{(\tau-t_o)(\tau t_o-1)}\right\}. \tag{18}$$

The lowest order of $s^{(n)}(t)$ appearing in Eqs.(15) and (16) is $s^{(o)}(t;\ t_o^{(o)})$ which may be taken from an appropriately chosen basic flow. The basic reference flow should be chosen as simple as practical and as effective. For example, one may choose an inclined flat plate, particularly when $\beta(s)$ is everywhere small. When $\beta(s)$ is moderate or large, as an alternative a two-sided polygon may be chosen with appropriate surface inclinations. For each iteration in Eqs.(15) and (16) with $n = 0, 1, 2, \ldots$, we let $\beta(s^{(n)}(t))$ assume the corresponding value of the prescribed $\beta(s)$ of the given profile with $s = s^{(n)}(t)$, this being possible when $A^{(n)}$ is so determined that each $s^{(n)}(t)$ satisfies Condition (11). As a further note, the problem of smooth detachment requires application of the same iteration procedure to the additional condition (12).

The above iteration method has been programmed and carried out on an IBM-7090 computer with a prescribed maximum allowable error for $|s^{(n+1)} - s^{(n)}|$ and $|t_o^{(n+1)} - t_o^{(n)}|$. From the actual performance in several specific cases as reported in Reference [2], the convergence of this iteration method has been found to be quite fast in all the cases tried. However, in view of considerable amount of complicated numerical integrations involved in these iterations, it calls for a high speed eletronic computer almost as a necessary tool.

We now introduce an approximate numerical scheme by which most of the complicated computations can be curtailed or avoided.

SINGLE-PARAMETER ITERATION PRINCIPLE - AN APPROXIMATE NUMERICAL SCHEME

The proposed approximate numerical scheme can be described by the following steps:

(i) For planar bodies with small camber, that is, $\beta(s)$ is everywhere small, we first neglect β in Eq.(9), giving for $s(t)$ an approximate value

$$s_*(t;t_o) = \int_{-1}^{t} \frac{df}{dt}\frac{dt}{t} = \left[\frac{f(t)}{t}\right]_{-1}^{t} + \int_{-1}^{t} \frac{f(t)}{t^2}\,dt \tag{19a}$$

which can be integrated in a closed form

$$s_*(t;t_o) + b = \frac{At}{(t-t_o)(t-\bar{t}_o)(t-t_o^{-1})(t-\bar{t}_o^{-1})} +$$

$$+ B\left[\frac{1}{t_o^2}\log(t-t_o) - \log\left(t-\frac{1}{t_o}\right)\right] + \bar{B}\left[\frac{1}{\bar{t}_o^2}\log(t-\bar{t}_o) - \log\left(t-\frac{1}{\bar{t}_o}\right)\right] \tag{19b}$$

where the coefficient B is given by Eq.(13b) (and hence proportional to constant A), and the real constant b is of such value as to make $s_* = 0$ at $t = -1$. The constant factor A on the right hand side of Eq.(19) is determined by again using Condition (11). The above function $s_*(t;\ t_o)$ has the same formal expression as that for the flat plate case except that now t_o is so far undetermined.

For the case in which the inclination angle $\beta(s)$ is no longer everywhere small, one may also at this stage approximate $s(t;\, t_o)$ by the corresponding solution of an adequately chosen symmetric wedge or an asymmetric wedge (a bent flat plate), the solution of these two cases being available in simple closed forms (see Reference [2]). These known solutions will also be denoted symbolically by $s_*(t;\, t_o)$, with t_o still being "floating".

(ii) We next approximate $\beta(\tau)$ in Eq.(10) by

$$\beta(t) \equiv \beta(s(t;\, t_o)) \approx \beta(s_*(t;\, t_o)) \,, \tag{20}$$

and then determine t_o from Eq.(10) by iteration until the error $|t_o^{(n+1)} - t_o^{(n)}|$ is within a prescribed limit.

(iii) With t_o so determined and $\beta(t)$ approximated by Eq.(20), $s(t;\, t_o)$ can now be calculated from Eq.(9), $C_p(t)$ from Eq.(14), the lift L and drag D from Eq.(13), and likewise the other flow quantities.

It may be noted that the most significant saving in computation efforts comes from the curtailment of the complicated iteration in Eq.(15) which involves a double integration each time around. With this simplification, the iteration is reduced to one with respect to a single complex parameter t_o which is now governed by Eq.(10) alone and for which the process involves only a single integration. In fact the numerical computation is so much simplified that, even without the facility of high speed eletronic computers, it becomes quite feasible to carry out this approximate numerical scheme on a hand computer. The success and the advantages of this approximate method have been demonstrated in a few specific applications already made, as compared with the corresponding exact solutions.

As a definite example, the circular arc hydrofoil of total arc angle 16° has been considered for a range of the incidence angle α and the cavitation number σ. The original integral iteration method has been programmed for the IBM-7090 computer, the accuracy of the numerical integration with respect to t in Eqs.(15), (16) being found satisfactory with 320 intervals between $t = -1$ and 1. For each prescribed α and σ, the computer time for a single iteration of Eqs.(15) and (16) using this program has been recorded to take about 30 seconds: the iteration continues until the error of t_o becomes less than 10^{-4}, the corresponding error of $s(t; t_o)$ appears to be even smaller in this case. By adopting the present approximate method, the result shows that the convergence of the iteration in calculating t_o is also very fast. As a consequence of the simplifications introduced, the total computer time for each iterative calculation of t_o is reduced to a mere 3 seconds -- a saving of 90% in computer time. The accuracy of this approximate method is clearly indicated by its comparison with the exact method of integral iteration, as shown in Table I. Both methods yield in this case very accurate results, which cannot be distinguished in the graphic plotting. The final results of the lift and drag coefficients of the circular arc are shown versus σ in Figures 2 and 3, in which Parkin's experimental data [3] are included for comparison. For small α the curve terminates at a certain critical value of σ at which the point C' in Fig. 1 approaches the trailing edge B, indicating the flow going through transition to the partially cavitating type [1] for further increase in σ. It is worth emphasizing that the accuracy of the present approximate method is uniformly valid in the angle of attack α, as can be noted from Table I.

It may also be pointed out here that the advantages of this approximate method can be fully utilized in computing the exact solution by simply transferring the numerical results of the approximate solution as the initial data into the program of the integral iteration scheme, as given by Eqs. (15) - (18), which is the logical scheme of the two methods.

ACKNOWLEDGMENTS

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TABLE I

Comparison of Original Integral Iteration and the Approximate Numerical Method Circular Arc Hydrofoil of Semi-arc Angle $\gamma = 8^{\circ}$

		Original Integral Iteration Method		Approximate Numerical Method	
α	σ	C_L	C_D	C_L	C_D
5°	0.01	0.28698	0.02100	0.28585	0.02079
	0.1	0.33436	0.02417	0.33335	0.02393
	0.18	0.40187	0.02883	0.40125	0.02856
	0.3	0.53344	0.03904	0.53346	0.03884
10°	0.01	0.37744	0.05805	0.37681	0.05777
	0.1	0.42222	0.06494	0.42159	0.06462
	0.2	0.49173	0.07576	0.49120	0.07540
	0.4	0.67804	0.10606	0.67805	0.10567
	0.53	0.82120	0.13057	0.82179	0.13033
15°	0.01	0.45080	0.10896	0.45061	0.10867
	0.1	0.49714	0.12029	0.49749	0.12010
	0.2	0.56135	0.13598	0.56190	0.13578
	0.4	0.72224	0.17636	0.72316	0.17614
	0.6	0.91408	0.22575	0.91570	0.22561
	0.7	1.01779	0.25302	1.01969	0.25295
20°	0.01	0.50741	0.17031	0.50748	0.17005
	0.1	0.55699	0.18705	0.55707	0.18668
	0.2	0.62013	0.20854	0.62026	0.20823
	0.4	0.76819	0.25959	0.76900	0.25939
	0.6	0.93968	0.31960	0.94102	0.31945
	0.8	1.12722	0.38630	1.12907	0.38624
	1.0	1.32524	0.45765	1.32747	0.45788
25°	0.01	0.54762	0.23907	0.54785	0.23884
	0.1	0.59912	0.26161	0.59963	0.26147
	0.2	0.66207	0.28938	0.66267	0.28923
	0.4	0.80348	0.35239	0.80434	0.35225
	0.6	0.96187	0.42372	0.96301	0.42358
	0.8	1.13281	0.50149	1.13427	0.50139
	1.0	1.31270	0.58407	1.31447	0.58406
30°	0.01	0.57201	0.31250	0.57237	0.31244
	0.1	0.62483	0.34143	0.62541	0.34135
	0.2	0.68751	0.37594	0.68817	0.37586
	0.4	0.82380	0.45154	0.82467	0.45146
	0.6	0.97237	0.53463	0.97352	0.53458
	0.8	1.13051	0.62380	1.13191	0.62379
	1.0	1.29581	0.71757	1.29749	0.71762

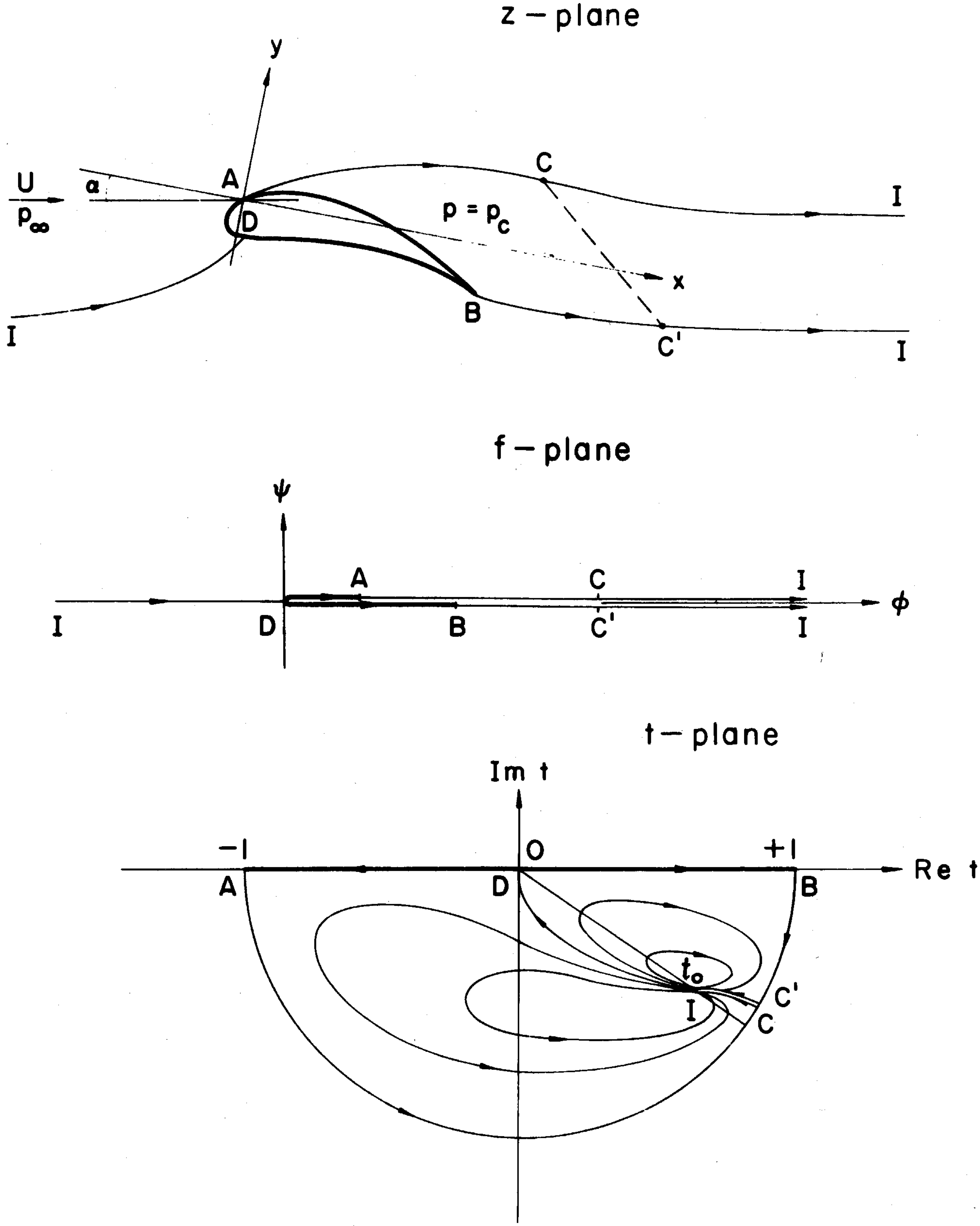

Fig. 1. The free-streamline model for the wake flow past a body of arbitrary profile and its conformal mapping planes.

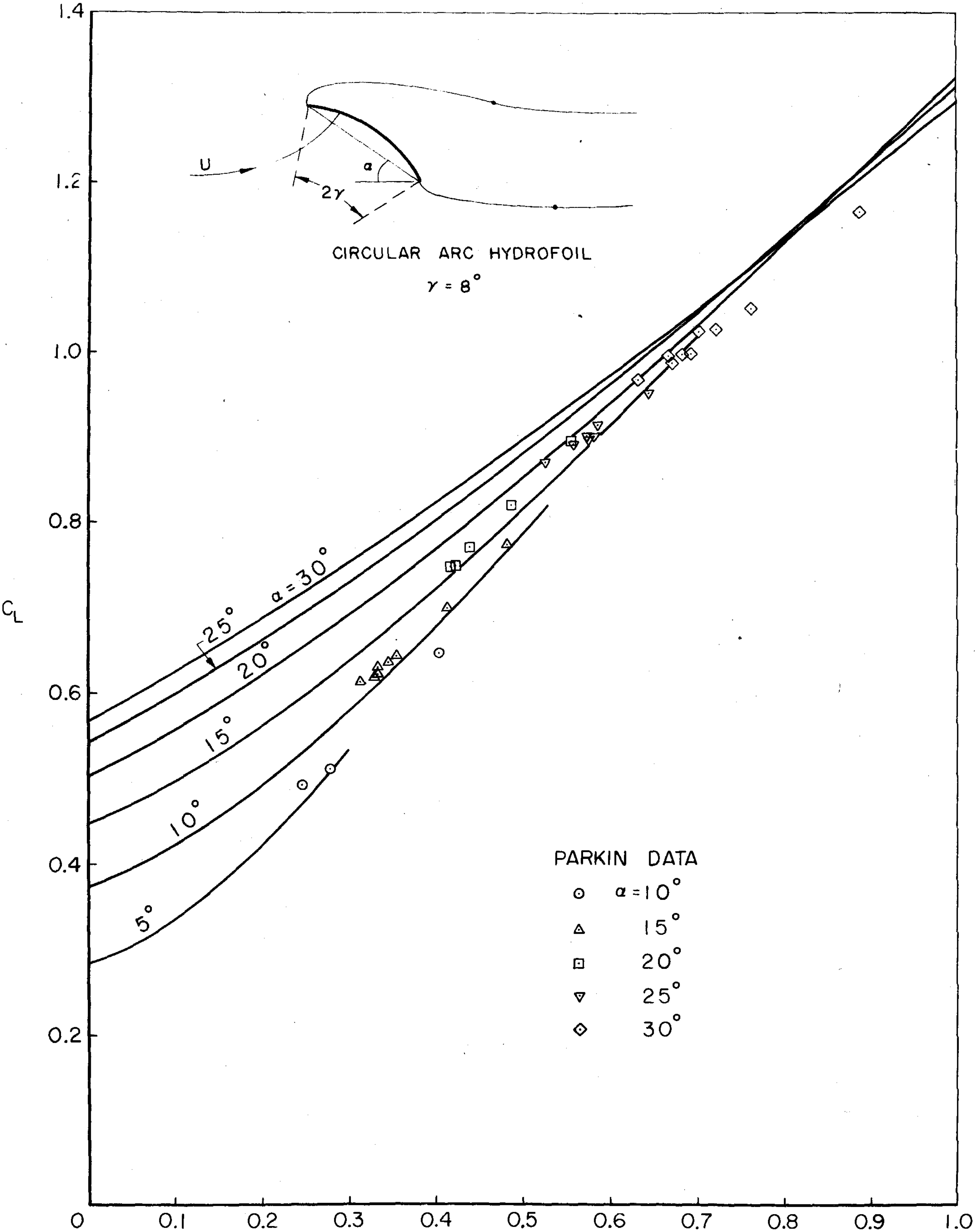

Fig. 2. Variation of C_L with the cavitation number σ for an inclined circular arc hydrofoil of semi-arc angle $\gamma = 8^\circ$.

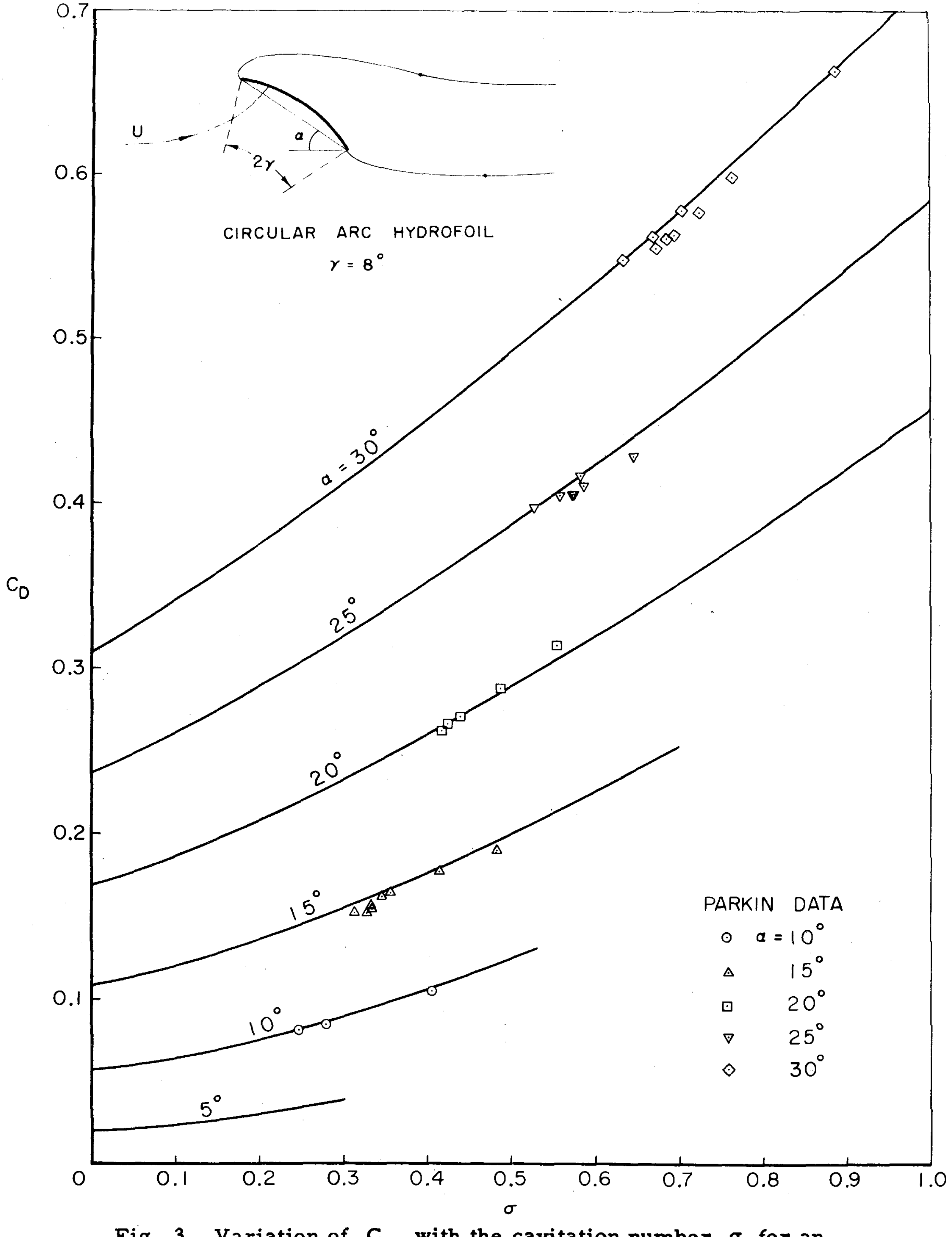

Fig. 3. Variation of C_D with the cavitation number σ for an inclined circular arc hydrofoil of semi-arc angle $\gamma = 8°$.

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